Sample Paper 2014: Paper 1

Question 9 (50 marks)

Question 9 (a)

(i) $f(x) = -0.5x^2 + 5x - 0.98$

$f(0.2) = -0.5(0.2)^2 + 5(0.2) - 0.98 = 0$

(ii) $f(x) = -0.5x^2 + 5x - 0.98$

$f'(x) = -x + 5 = 0 \Rightarrow x = 5$

$f(5) = -0.5(5)^2 + 5(5) - 0.98 = 11.52 \Rightarrow (5, 11.52)$ is a turning point.

$f''(x) = -1 < 0 \Rightarrow (5, 11.52)$ is a local maximum.

Question 9 (b) (i)

$f(x) = -0.5x^2 + 5x - 0.98$

$f(1) = -0.5(1)^2 + 5(1) - 0.98 = 3.52$

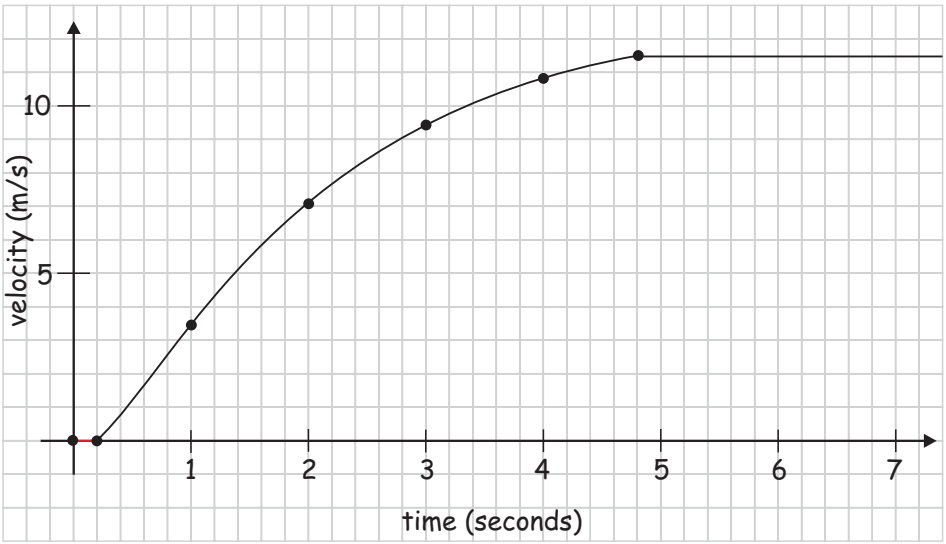
$f(2) = -0.5(2)^2 + 5(2) - 0.98 = 7.02$

$f(3) = -0.5(3)^2 + 5(3) - 0.98 = 9.52$

$f(4) = -0.5(4)^2 + 5(4) - 0.98 = 11.02$

$f(5) = -0.5(5)^2 + 5(5) - 0.98 = 11.52$

[Work out a number of points for the middle part of the graph and then plot it.]



Question 9 (b) (ii)

In the first 0·2 s the sprinter does not move. He/she is still in the blocks. To find the distance travelled over the first 5 s find the area under the velocity curve from 0·2 s to 5 s.

$$\boxed{v = \frac{ds}{dt} \Rightarrow s = \int v\,dt}$$

$$s = \int v\,dt$$

$$= \int (-0.5t^2 + 5t - 0.98)\,dt$$

$$= \frac{-0.5t^3}{3} + \frac{5t^2}{2} - 0.98t + c$$

To find the distance s travelled replace t by 5 and then t by 0·2 and subtract the answers.

$$s=\left[\frac{-0.5(5)^3}{3}+\frac{5(5)^2}{2}-0.98(5)+c\right]-\left[\frac{-0.5(0.2)^3}{3}+\frac{5(0.2)^2}{2}-0.98(0.2)+c\right]=36.9\text{ m}$$

Question 9 (b) (iii)

The sprinter runs 36·9 m over the first 5 seconds.
He/she runs the rest of the race (63·1 m) at the maximum speed of 11·52 m/s.

$$11.52=\frac{63.1}{t}\Rightarrow t=\frac{63.1}{11.52}=5.48\text{ s}\qquad \boxed{v=\frac{s}{t}}$$

Answer: Total time = 10·48 s

Question 9 (c)

(i)

$$\frac{dV}{dt}\propto -A\Rightarrow\frac{dV}{dt}=-kA$$

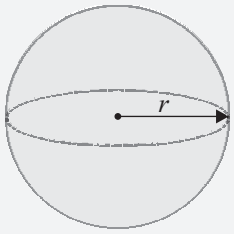
$$\frac{d(\frac{4}{3}\pi r^3)}{dt}=-k(4\pi r^2)$$

$$\tfrac{4}{3}\pi r^2\frac{dr}{dt}=-k(4\pi r^2)$$

$$\frac{dr}{dt}=-k$$

Formulae and Tables Book

Surface area and volume:
Sphere [page 8]



$A=4\pi r^2$

$V=\frac{4}{3}\pi r^3$

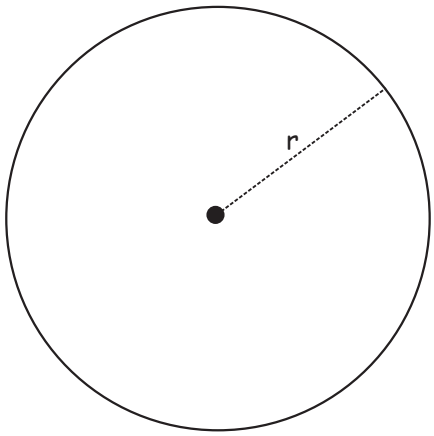
(ii)

$$\frac{dr}{dt}=-k\Rightarrow\int dr=-k\int dt$$

$$r=-kt+c$$

$$t=0: r=r_0\Rightarrow c=r_0$$

$$\therefore r=-kt+r_0$$



$$t=1: \tfrac{2}{3}\pi r_0^2=\tfrac{4}{3}\pi r^3\Rightarrow r=\frac{r_0}{2^{\frac{1}{3}}}$$

$$\therefore\frac{r_0}{2^{\frac{1}{3}}}=-k(1)+r_0\Rightarrow k=\left(1-\frac{1}{2^{\frac{1}{3}}}\right)r_0$$

$$t=T: r=0\Rightarrow 0=-kT+r_0$$

$$\therefore T=\frac{r_0}{k}=\frac{r_0}{\left(1-\frac{1}{2^{\frac{1}{3}}}\right)r_0}=4.847\text{ hours}\approx 291\text{ minutes}$$